MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2018

16-04-2018 MORNING (ONLINE)

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with only Black Ball Point Pen provided in the examination hall.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of 3 hours duration.
4. The Test Booklet consists of 90 questions. The maximum marks are 360.
5. There are three parts in the question paper A, B, C consisting of **Physics, Chemistry and Mathematics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.
6. Candidates will be awarded marks as started above in instruction No. 5 for correct response of each question. ¼ (one fourth) marks of the total marks allotted to the question (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from that total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. For writing particulars / marking responses on Side–1 and Side–2 of the Answer Sheet use only Black Ball Point Pen provided in the examination hall.
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in four pages at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. However, the candidates are allowed to take away this Test Booklet with them.
12. The CODE for this Booklet is D. Make sure that the CODE printed on Side–2 of the Answer Sheet is same as that on this Booklet. Also tally the serial number of Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet

PART-A-PHYSICS

1. The percentage errors in quantities P, Q, R and S are 0.5%, 1%, 3% and 1.5% respectively in the measurement of a physical quantity A = $\frac{P^3Q^2}{\sqrt{R}\,S}$

The maximum percentage error in the value of A will be :

(1) 6.0% (2) 7.5% (3) 8.5% (4*) 6.5%

Sol. $\frac{\Delta A}{A} = \frac{3\Delta P}{P} + \frac{2\Delta Q}{Q} + \frac{1}{2}\frac{\Delta R}{R} + \frac{\Delta S}{S}$

$= 3 \times 0.5 + 2 \times 1 + \frac{1}{2} \times 3 + 1.5$

= 1.5 + 2 + 1.5 + 1.5

$\frac{\Delta A}{A} = 6.5\%$

2. Let $\vec{A} = (\hat{i} + \hat{j})$ and, $\vec{B} = (2\hat{i} - \hat{j})$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A}\cdot\vec{C} = \vec{B}\cdot\vec{C} = \vec{A}\cdot\vec{B}$, is given by :

(1) $\sqrt{\frac{10}{9}}$ (2*) $\sqrt{\frac{5}{9}}$ (3) $\sqrt{\frac{20}{9}}$ (4) $\sqrt{\frac{9}{12}}$

Sol. If $\vec{C} = a\hat{i} + b\hat{j}$ then

$\vec{A}\cdot\vec{C} = \vec{A}\cdot\vec{B}$ -

a + b = 1 ....(i)

$\vec{B}\cdot\vec{C} = \vec{A}\cdot\vec{B}$

2a – b = 1 ....(ii)

Solving equation (i) and (ii) we get

$a = \frac{1}{3}, b = \frac{2}{3}$

$|\vec{C}| = \sqrt{\frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{5}{9}}$

3. A body of mass m starts moving from rest along x-axis so that its velocity varies as v = a $\sqrt{s}$ where a is a constant and s is the distance covered by the body. The total work done by all the forces acting on the body in the first t seconds after the start of the motion is :

(1*) $\frac{1}{8}$ m a^4 t^2 (2) 8 m a^4 t^2 (4) 4 m a^4 t^2 (4) $\frac{1}{4}$ m a^4 t^2

Sol. $v = a\sqrt{s} = \frac{ds}{dt}$

$2\sqrt{s} = at$

$S = \frac{a^2t^2}{4}$

$F = m \times \frac{a^2}{2}$

Work $= \frac{ma^2}{2} \times \frac{a^2t^2}{4} = \frac{1}{8}ma^4t^2$

4. Two particles of the same mass m are moving in circular orbits because of force, given by $F(r) = \frac{-16}{r} - r^3$. The first particle is at a distance r = 1, and the second, at r = 4. The best estimate for the ratio of kinetic energies of the first and the second particle is closest to :

(1*) 6×10^{-2} (2) 3×10^{-3} (3) 10^{-1} (4) 6×10^{2}

Sol. $\frac{mV^2}{r} = \frac{16}{r} + r^3$

$KE_0 = \frac{1}{2}mV^2$

$= \frac{1}{2}[16 + r^4]$

$\frac{K_1}{K_2} = \frac{\frac{16+1}{2}}{\frac{16+256}{2}} = \frac{17}{272}$

$\frac{K_1}{K_2} \approx 6 \times 10^{-2}$

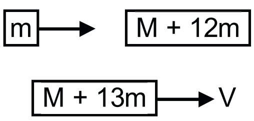
5. An oscillator of mass M is at rest in its equilibrium position in a potential V = $\frac{1}{2}$ k(x – X)^2. A particle of mass m comes from right with speed u and collides completely inelastically with M and sticks to it. This process repeats every time the oscillator crosses its equilibrium position. The amplitude of oscillations after 13 collisions is : (M = 10, m = 5, u = 1, k = 1)

(1*) $\frac{1}{\sqrt{3}}$ (2) $\frac{1}{2}$ (3) $\frac{2}{3}$ (4) $\sqrt{\frac{3}{5}}$

Sol. In first collision mu momentum will be imparted to system. In second collision when momentum of (M + m) is in opposite direction mu momentum of particle will make its momentum zero.

on 13th collision

mu = (M + 13m)v



$v = \frac{mu}{M + 13m} = \frac{u}{15}$

v = ωA

$$\Rightarrow \quad \frac{u}{15} = \sqrt{\frac{K}{M+13m}} \times A \quad \Rightarrow \quad A = \frac{1}{15}\sqrt{\frac{75}{1}} = \frac{1}{\sqrt{3}}$$

6. Suppose that the angular velocity of rotation of earth is increased. Then, as a consequence :

(1) Weight of the object, everywhere on the earth, will increase.

(2) Weight of the object, everywhere on the earth, will decrease.

(3) There will be no change in weight anywhere on the earth.

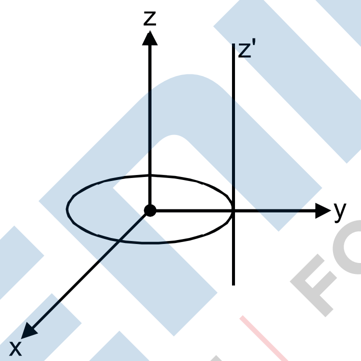
(4*) Except at poles, weight of the object on the earth will decrease

Sol. $g' = g - \omega^2 R\cos^2\phi$

Where ϕ is latitude there will be no change in gravity at poles as $\phi = 90°$

At all other points as ω increases g' will decrease.

7. A thin circular disk is in the xy plane as shown in the figure. The ratio of its moment of inertia about z and z' axes will be :



(1*) 1 : 3 (2) 1 : 4 (3) 1 : 5 (4) 1 : 2

Sol. $I_z = \frac{mR^2}{3}$

$I_z' = \frac{3}{2}mR^2 = \frac{I_z}{I_z'} = \frac{1}{3}$

8. The relative uncertainty in the period of a satellite orbiting around the earth is 10^{-2}. If the relative uncertainty in the radius of the orbit is negligible, the relative uncertainty in the mass of the earth is :

(1) 10^{-2} (2*) 2×10^{-2} (3) 3×10^{-2} (4) 6×10^{-2}

Sol. From kepler's law

$$T^2 = \frac{4\pi^2}{GM} r^3$$

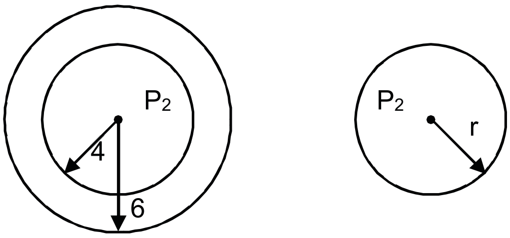
$$\left|\frac{\Delta M}{M}\right| = 2\frac{\Delta T}{T} = 2 \times 10^{-3}$$

9. A small soap bubble of radius 4 cm is trapped inside another bubble of radius 6 cm without any contact. Let P_2 be the pressure inside the inner bubble and P_0, the pressure outside the outer bubble. Radius off another bubble with pressure difference $P_2 - P_0$ between its inside and outside would be :

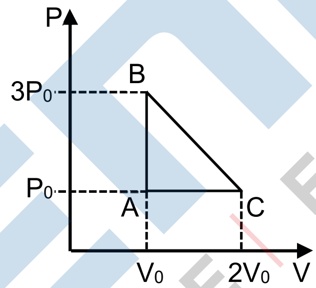
(1) 12 cm (2*) 2.4 cm (3) 6 cm (4) 4.8 cm

Sol. $P_2 = P_0 + \frac{4T}{6} + \frac{4T}{4}$ $\quad P_2 = P_0 + \frac{4T}{r}$

$\Rightarrow \quad \frac{1}{r} = \frac{1}{6} + \frac{1}{4} \quad \Rightarrow \quad r = 2.4$ cm



10. One mole of an ideal monoatomic gas is taken along the path ABCA as shown in the PV diagram. The maximum temperature attained by the gas along the patch BC is given by :



(1) $\frac{25}{16}\frac{P_0V_0}{R}$ (2*) $\frac{25}{8}\frac{P_0V_0}{R}$ (3) $\frac{25}{4}\frac{P_0V_0}{R}$ (4) $\frac{5}{8}\frac{P_0V_0}{R}$

Sol. Equation of line BC

$$P = P_0 - \frac{2P_0}{V_0}(V - 2V_0)$$

$$\text{Temperature} = \frac{P_0V - \frac{2P_0V^2}{V_0} + 4P_0V}{1 \times R}$$

$$T = \frac{P_0}{R}\left[5V - \frac{2V^2}{V_0}\right]$$

$$\frac{dT}{dV} = 0 \quad \Rightarrow \quad 5 - \frac{4V}{V_0} = 0 \quad \Rightarrow \quad V = \frac{5}{4}V_0$$

$$T = \frac{P_0}{R}\left[5 \times \frac{5V_0}{4} - \frac{2}{V_0} \times \frac{25}{16}V_0^2\right]$$

$T = \frac{25}{8}\frac{P_0V_0}{R}$

11. Two moles of helium are mixed with n moles of hydrogen. If $\frac{C_P}{C_V} = \frac{3}{2}$ for the mixture, then the value of n is :

(1) 1 (2) 3 (3*) 2 (4) 3/2

Sol. $\frac{C_p}{C_v} = \frac{f_{mix}+2}{f_{mix}} = \frac{3}{2}$

$\Rightarrow$ $f_{mix} = 4$

$f_{mix} = \frac{n_1f_1 + n_2f_2}{n_1 + n_2}$

$\Rightarrow$ $\frac{4 - 2\times 3 + n_2 \times 5}{2 + n_2}$ $\Rightarrow$ n_2 = 2mole

12. A particle executes simple harmonic motion and is located at x = a, b and c at times t_o, $2t_0$ and $3t_0$ respectively. The frequency of the oscillation is :

(1*) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$ (2) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{2c}\right)$

(3) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{2a+3c}{b}\right)$ (4) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+2b}{3c}\right)$

Sol. $a = A\sin\omega t_0$

$b = A\sin 2\omega t_0$

$c = A\sin 3\omega t_0$

$a + c = A[\sin\omega t_0 + \sin 3\omega t_0] = 2A\sin 2\omega t_0\cos\omega t_0$

$\frac{a+c}{b} = 2\cos\omega t_0$

$\omega = \frac{1}{t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$ $\Rightarrow$ $f = \frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$

13. Two sitar strings, A and B, playing the note 'Dha' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B is lightly increased and the beat frequency is found to decrease by 3 Hz. If the frequency of A is 425 Hz, the original frequency of B is :

(1) 430 Hz (2*) 420 Hz (3) 428 Hz (4) 422 Hz

Sol. Frequency of B is either 420Hz or 430Hz As tension in B is increased its frequency will increase.

If frequency is 430Hz, beat frequency will increase

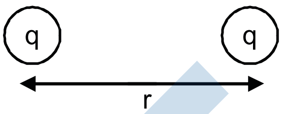
If frequency is 420 Hz beat frequency will decrease, hence correct answer is 420Hz

14. Two identical conducting spheres A and B, carry equal charge. They are separated by a distance much larger than their diameters, and the force between them is F. A third identical conducting sphere, C, is uncharged. Sphere C is first touched to A, then to B, and then removed. As a result, the force between A and B would be equal to :

(1) F (2) $\frac{3F}{4}$ (3*) $\frac{3F}{8}$ (4) $\frac{F}{2}$

Sol. $F = \frac{kq^2}{r^2}$ when A and C are touched charge on both will be $\frac{q}{2}$

Then when B and C are touched



$$q_B = \frac{\frac{q}{2}+q}{2} = \frac{3q}{4}$$

$$F' = \frac{kq_A + q_B}{r^2} = \frac{k \times \frac{q}{2} \times \frac{3q}{4}}{r^2} = \frac{3}{8}\frac{kq^2}{r^2} = \frac{3}{8}F$$

15. A heating element has a resistance of 100 Ω at room temperature. When it is connected to a supply of 220 V, a steady current of 2 A passes in it and temperature is 500°C more than room temperature. What is the temperature coefficient of resistance of the height element?

(1) 0.5×10^{-4} °C^{-1} (2) 5×10^{-4} °C^{-1} (3) 1×10^{-4} °C^{-1} (4*) 2×10^{-4} °C^{-1}

Sol. Resistance after temperature increases by $500°C = \frac{220}{2} = 110\Omega$

$110 = 100 (1 + \alpha\, 500)$

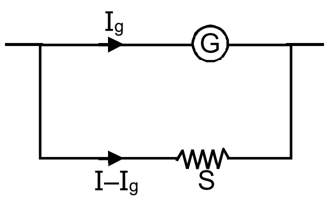
$$\alpha = \frac{10}{100 \times 500}$$

$\alpha = 2 \times 10^{-4}$ °C^{-1}

16. A galvanometer with its coil resistance 25Ω requires a current of 1 mA for its full deflection. In order to construct an ammeter to read up to a current of 2A, the approximate value of the shunt resistance should be :

(1) 2.5×10^{-3} Ω (2*) 1.25×10^{-2} Ω (3) 1.25×10^{-3} Ω (4) 2.5×10^{-2} Ω

Sol. $I_gR_g = (I - I_g)S$

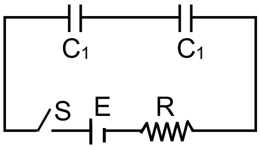


$$S \approx \frac{10^{-3} \times 25}{2}$$

$S \approx 12.5 \times 10^{-3}$

or 1.25×10^{-2} Ω

17. In the following circuit, the switch S is closed at t = 0. The charge on the capacitor C_1 as a function of time will be given $\left(C_{eq} = \frac{C_1C_2}{C_1+C_2}\right)$

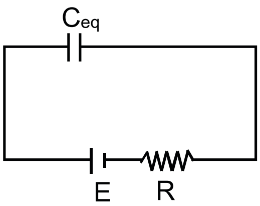


(1) $C_1E\,[1 - \exp(-tR/C_1)]$ (2) $C_2E\,[1 - \exp(-t/RC_2)]$

(3*) $C_{eq}E\,[1 - \exp(-t/RC_{eq})]$ (4) $C_{eq}E\,\exp(-t/RC_{eq})]$

Sol. $q = C_{eq}E\left[1 - e^{-t/Rc_{eq}}\right]$

Both capacitor will have same charge as they are connected in series.



18. A coil of cross-sectional area A having n turns is placed in a uniform magnetic field B. When it is rotated with an angular velocity ω, the maximum e.m.f. induced in the coil will be :

(1) $3\,nBA\omega$ (2) $\frac{3}{2}nBA\omega$ (3*) $nBA\omega$ (4) $\frac{1}{2}nBA\omega$

Sol. $\varepsilon = BA\omega n \sin\omega t$

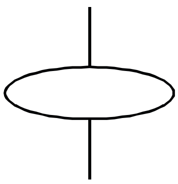
$\varepsilon_{max} = BA\omega n$

19. A charge q is spread uniformly over an insulated loop of radius r. If it is rotated with an angular velocity ω with respect to normal axis then the magnetic moment of the loop is :

(1) $q\,\omega r^2$ (2) $\frac{4}{3}q\,\omega r^2$ (3) $\frac{3}{2}q\,\omega r^2$ (4*) $\frac{1}{2}q\,\omega r^2$

Sol. $\frac{M}{L} = \frac{q}{2m}$

$M = \frac{q}{2m} \times mr^2\omega$



$M = \frac{q\omega r^2}{2}$

20. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns, giving the output power at 230 V. If the current in the primary of the transformer is 5 A, and its efficiency is 90%, the output current would be :

(1) 50 A (2*) 45 A (3) 25 A (4) 20 A

Sol. Efficiency $n = 0.9 = \frac{P_s}{P_P}$

$V_sI_s = 0.9 \times V_PI_P$

$I_s = \frac{0.9 \times 2300 \times 5}{230} = 45A$

21. A plane electromagnetic wave of wavelength λ has an intensity I. It is propagating along the positive Y-direction. The allowed expressions for the electric and magnetic fields are given by :

(1*) $\vec{E} = \sqrt{\frac{2I}{\varepsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y - ct)\right]\hat{k}; \vec{B} = +\frac{1}{c}E\hat{i}$

(2) $\vec{E} = \sqrt{\frac{2I}{\varepsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y + ct)\right]\hat{k}; \vec{B} = \frac{1}{c}E\hat{i}$

(3) $\vec{E} = \sqrt{\frac{I}{\varepsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y - ct)\right]\hat{k}; \vec{B} = +\frac{1}{c}E\hat{i}$

(4) $\vec{E} = \sqrt{\frac{I}{\varepsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y - ct)\right]\hat{i}; \vec{B} = \frac{1}{c}E\hat{k}$

Sol. If E_0 is magnitude of electric field then $\frac{1}{2}\varepsilon_0 E^2 \times C = I$

$E_0 = \sqrt{\frac{2I}{C\varepsilon_0}}$

$B_0 = \frac{E_0}{C}$

direction of $\vec{E} \times \vec{B}$ will be along $+\hat{j}$.

22. A ray of light is incident at an angle of 60° on one face of a prism of angle 30°. The emergent ray of light makes an angle of 30° with incident ray. The angle made by the emergent ray with second face of prism will be :

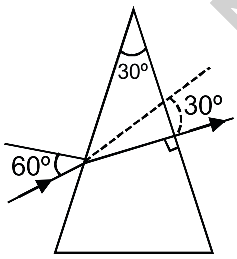
(1) 0° (2*) 90° (3) 45° (4) 30°

Sol. δ = I + e – A

30 = 60 + E – 30º

$\Rightarrow$ E = 0

So angle with face = 90º



23. Unpolarized light of intensity I is incident on a system of two polarizers, A followed by B. The intensity of emergent light is I/2. If a third polarizer C is placed between A and B, the intensity of emergent light is reduced to I/3. The angle between the polarizers A and C is θ. Then :

(1) $\cos\theta = \left(\frac{2}{3}\right)^{1/2}$ (2*) $\cos\theta = \left(\frac{2}{3}\right)^{1/4}$ (3) $\cos\theta = \left(\frac{1}{3}\right)^{1/2}$ (4) $\cos\theta = \left(\frac{1}{3}\right)^{1/4}$

Sol. A and B have same alignment of transmission axis.

Lets assume c is introduced at θ angle

$$\frac{I}{2}\cos^2\theta \times \cos^2\theta = \frac{I}{3}$$

$$\cos^4\theta = \frac{2}{3} \quad \Rightarrow \quad \cos\theta = \left(\frac{2}{3}\right)^{\frac{1}{4}}$$

24. The de-Broglie wavelength (λ_B) associated with the electron orbiting in the second excited state of hydrogen atom is related to that in the ground state (λ_G) by :

(1) $\lambda_B = 2\lambda_G$ (2*) $\lambda_B = 3\lambda_G$ (3) $\lambda_B = \lambda_{G/2}$ (4) $\lambda_B = \lambda_{G/3}$

Sol. $\frac{\lambda_B}{\lambda_G} = \frac{P_a}{P_B} = \frac{mv_G}{mv_B}$

$V \times \frac{z}{n}$ So $\frac{\lambda_B}{\lambda_G} = \frac{n_B}{n_G} = \frac{3}{1}$

$\lambda_B = 3\lambda_G$

Length of orbit = n × λ

$$\lambda = \frac{2\pi r}{n} \quad \Rightarrow \quad \lambda \propto \frac{1}{n}$$

25. Both the nucleus and the atom of some element are in their respective first excited states. They get de-excited by emitting photons of wavelengths λ_N, λ_A respectively. The ratio $\frac{\lambda_N}{\lambda_A}$ is closest to :

(1*) 10^{-6} (2) 10 (3) 10^{-10} (4) 10^{-1}

Sol. $\frac{\lambda_N}{\lambda_a} = \frac{E_a}{E_N}$

where E_a and E_N are energies of photons from atom and nucleus respectively. E_N is of the order of MeV and E_a in few eV.

So $\frac{\lambda_N}{\lambda_a} = 10^{-6}$

26. At some instant, a radioactive sample S_1 having an activity 5 μCi has twice the number of nuclei as another sample S_2 which has an activity of 10 μC_i. The half lives of S_1 and S_2 are :

(1) 20 years and 5 years, respectively (2) 20 years and 10 years, respectively

(3*) 5 years and 20 years, respectively (4) 10 years and 20 years, respectively

Sol. Given : $N_1 = 2N_2$

$$\lambda_1 N_1 = \frac{\ln 2}{t_1} \times N_1 = 5\mu c_1$$

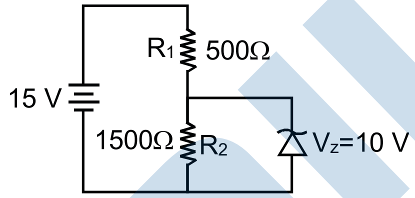
$$\lambda_2 N_2 = \frac{\ln 2}{t_2} \times N_2 = 10\mu c_2$$

$$\frac{t_2}{t_1} \times \frac{N_1}{N_2} = \frac{1}{2}$$

$$\frac{t_2}{t_1} = \frac{1}{4}$$

Hence 5 years and 20 years

27. In the given circuit, the current through zener diode is :



(1) 5.5 mA (2) 6.7 mA (3) 2.5 mA (4*) 3.3 mA

Sol. Current in $R_1 = I_1 = \frac{5}{500}$

$I_1 = 10$ mA

Current in $R_2 = I_2 = \frac{10}{1500} \Rightarrow I_2 = \frac{20}{3}\text{mA}$

Current in Zener diode $I_1 - I_2 = \left(10 - \frac{20}{3}\right)\text{mA} = \frac{10}{3}\text{mA}$

28. A carrier wave of peak voltage 14 V is used for transmitting a message signal. The peak voltage of modulating signal given to achieve a modulation index of 80% will be :

(1) 7 V (2) 28 V (3*) 11.2 V (4) 22.4 V

Sol. $m = \frac{A_m}{A_c}$

$A_m = 0.8 \times 14$

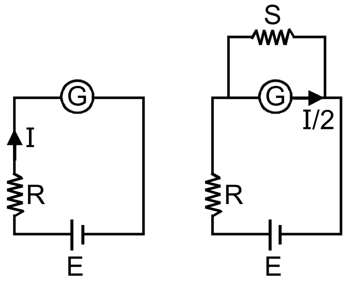
$= 11.2$ V

29. In a circuit for finding the resistance of a galvanometer by half deflection method, a 6 V battery and a high resistance of 11 kΩ are used. The figure of merit of the galvanometer is 60 μA/division. In the absence of shunt resistance, the galvanometer produces a deflection of θ = 9 divisions when current flows i the circuit. The value of the shunt resistance that can cause the deflection of $\theta/2$, is closest to :

(1) 550 Ω (2) 220 Ω (3) 55 Ω (4*) 110 Ω

Sol. $I = \frac{\varepsilon}{R+G}$ $G = \frac{1}{9} K\Omega$

$$\frac{I}{2} = \frac{\varepsilon}{R + \frac{GS}{G+S}} \times \frac{S}{S+G} \Rightarrow \frac{I}{2} = \frac{\varepsilon S}{R(S+G)+GS}$$



$$S = \frac{RG \times \frac{I}{2}}{\varepsilon - \frac{(R+G)I}{2}}$$

$$S = \frac{11 \times 10^3 \times \frac{1}{9} \times 10^3 \times 270 \times 10^{-6}}{6 - \left(\frac{6}{2}\right)}$$

S = 110 Ω

30. The end correction of a resonance column is 1 cm. If the shortest length resonating with the tuning fork is 10 cm, the next resonating length should be :

(1) 28 cm (2*) 32 cm (3) 36 cm (4) 40 cm

Sol. Given : e = 1 cm

For first resonance

$$\frac{\lambda}{4} = \ell_1 + e = 11 cm$$

For second resonance

$$\frac{3\lambda}{4} = \ell_1 + e \Rightarrow \ell_2 = 3 \times 11 - 1 = 32 cm$$

PART-B-CHEMISTRY

31. An unknown chlorohydrocarbon has 3.55% of chlorine. If each molecule of the hydrocarbon has one chlorine atom only; chlorine atoms present in 1 g of chlorohydrocarbon are :

(Atomic wt. of Cl=35.5 u; Avogadro constant = $6.023 \times 10^{23}\ mol^{-1}$)

(1*) 6.023×10^{20} (2) 6.023×10^{9} (3) 6.023×10^{21} (4) 6.023×10^{23}

Sol. CxYyCl

% Cl = 3.55

$$\text{Weight of Cl} = 1 \times \frac{3.55}{100}$$

$$n_{Cl^-} = \frac{1 \times 3.55}{100 \times 35.5}$$

$$\text{No of } Cl^- \text{ ion} = \frac{1 \times 3.55}{100 \times 35.5} \times 6.023 \times 10^{23}$$

$= 6.023 \times 10^{20}$

32. The gas phase reaction $2NO_2(g) \rightarrow N_2O_4(g)$ is an exothermic reaction. The decomposition of N_2O_4, in equilibrium mixture of $NO_2(g)$ and $N_2O_4(g)$, can be increased by :

(1) lowering the temperature. (2) increasing the pressure.

(3) addition of an inert gas at constant volume. (4*) addition of an inert gas at constant pressure.

Sol. $2NO_2(g) \longrightarrow N_2O_4(g)$ $\Delta H = (-)$

By addition of an inert gas at constant pressure, volume increases, so reaction moving in backward direction and decomposition of N_2O_4 increases.

33. Assuming ideal gas behaviour, the ratio of density of ammonia to that of hydrogen chloride at same temperature and pressure is : (Atomic wt. of Cl=35.5 u)

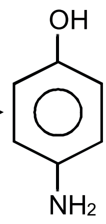
(1) 1.46 (2*) 0.46 (3) 1.64 (4) 0.64

Sol. $$d = \frac{P(M.w.)}{RT}$$

$$\frac{d_{NH_3}}{d_{HCl}} = \frac{(M.w.)_{NH_3}}{(M.w.)_{HCl}} = \frac{17}{36.5} = 0.46$$

34. When 9.65 ampere current was passed for 1.0 hour into nitrobenzene in acidic medium, the amount of p-aminophenol produced is :

(1*) 9.81 g (2) 10.9 g (3) 98.1 g (4) 109.0 g

Sol. $C_6H_5NO_2 \longrightarrow$ 

$$4e^- + 4H^+ + C_6H_5NO_2 \longrightarrow \underset{M.W.=109g}{C_6H_4(OH)(NH_2)} + H_2O$$

(v.f.) = 4 $\qquad W = ZIt = \frac{E}{F} \times I \times t \qquad \left(E = \frac{M}{4}\right)$

$$W = \frac{109 \times 9.65 \times 60 \times 60}{4 \times 96500}$$

W = 9.81 g

35. For which of the following processes, ΔS is negative ?

(1) $H_2(g) \rightarrow 2H(g)$

(2*) $N_2(g, 1\ atm) \rightarrow N_2(g, 5\ atm)$

(3) C(diamond) $\rightarrow$ C(graphite)

(4) $N_2(g, 273\ K) \rightarrow N_2(g, 300\ K)$

Sol. $N_2(g, 1\ atm) \longrightarrow N_2(g, 5\ atm)$

$\Delta S = \left(nC_p \ln\frac{T_2}{T_1}\right) + nR\ln\frac{V_2}{V_1}$ for isothermal process $T_1 = T_2$ and $\frac{V_2}{V_1} = \frac{P_1}{P_2}$

$$= 0 + nR\ln\frac{P_1}{P_2} = nR\ln\frac{1}{5}$$

36. Which one of the following is not a property of physical adsorption ?

(1) Higher the pressure, more the adsorption

(2) Lower the temperature, more the adsorption

(3) Greater the surface area, more the adsorption

(4*) Unilayer adsorption occurs

Sol. Physical adsorption is multilayer adsorption.

37. If 50% of a reaction occurs in 100 second and 75% of the reaction occurs in 200 second, the order of this reaction is :

(1) Zero (2*) 1 (3) 2 (4) 3

Sol.

C_o
$\downarrow$ 100 second
$\frac{C_o}{2}$
$\downarrow$ 200 second
$\frac{C_o}{4}$

75 % complete

First order reaction as half life is constant.

38. Which of the following statements is false ?

(1) Photon has momentum as well as wavelength.

(2) Splitting of spectral lines in electrical field is called Stark effect.

(3) Rydberg constant has unit of energy.

(4) Frequency of emitted radiation from a black body goes from a lower wavelength to higher wavelength as the temperature increases.

Ans. (2) and (4) [both are false]

Sol. When temperature is increased, black body emit high energy radiation, from higher wavelength to lower wavelength.

Rydberg constant has unit length^{-1} (i.e. cm^{-1})

39. At 320 K, a gas A_2 is 20% dissociated to A(g). The standard free energy change at 320 K and 1 atm in J mol^{-1} is approximately : (R = 8.314 JK^{-1} mol^{-1}; ln 2 = 0.693; ln 3 = 1.098)

(1) 4763 (2) 2068 (3) 1844 (4*) 4281

Sol. $A_2(g) \rightleftharpoons 2A(g)$

1 0

$1-1\times\frac{20}{100}$ $2\times\frac{20}{100}$

0.8 0.4

$$K_p = \frac{(p_A)^2}{(p_{A_2})} = \frac{0.4\times0.4}{0.8} = 0.2$$

$\Delta G^o = -2.303 \times 8.314 \times 320 \log_{10} 0.2 = 4281$ J/mole

40. The mass of a non-volatile, non-electrolyte solute (molar mass = 50 g mol^{-1}) needed to be dissolved in 114 g octane to reduce its vapour pressure to 75%, is :

(1) 37.5 g (2) 75 g (3) 150 g (4) 50 g

Ans. Bonus

Sol. $$\frac{P^0 - P_s}{P_s} = \frac{n}{N}$$

$$\frac{100P - 75P}{75P} = \frac{\frac{w}{50}}{1}$$

$$\frac{25}{75} = \frac{w}{50}$$

$$w = \frac{50}{3}g$$

41. The incorrect statement is :

(1) Cu^{2+} salts give red coloured borax bead test in reducing flame.

(2*) Cu^{2+} and Ni^{2+} ions give black precipitate with H_2S in presence of HCl solution.

(3) Ferric ion gives blood red colour with potassium thiocyanate.

(4) Cu^{2+} ion gives chocolate coloured precipitate with potassium ferrocyanide solution.

Sol. Due to common ion effect, sufficient S^{2-} concentration not produce and not formed ppt of NiS.

42. The incorrect geometry is represented by :

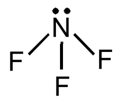
(1) BF_3 - trigonal planar

(2) H_2O - bent

(3*) NF_3 - trigonal planar

(4) AsF_5 - trigonal bipyramidal

Sol. NF_3



sp^3
Trigonal pyramidal

43. In Wilkinson's catalyst, the hybridization of central metal ion and its shape are respectively :

(1) sp^3d, trigonal bipyramidal

(2) sp^3, tetrahedral

(3*) dsp^2, square planar

(4) d^2sp^3, octahedral

Sol. Wilkinson catalyst

$[RhCl(PPh_3)_3]$

44. Among the oxides of nitrogen :

N_2O_3, N_2O_4 and N_2O_5 ; the molecule(s) having nitrogen-nitrogen bond is / are :

(1) Only N_2O_5

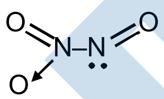
(2) N_2O_3 and N_2O_5

(3) N_2O_4 and N_2O_5

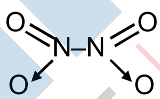
(4*) N_2O_3 and N_2O_4

Sol.

N_2O_3



N_2O_4



45. Which of the following complexes will show geometrical isomerism ?

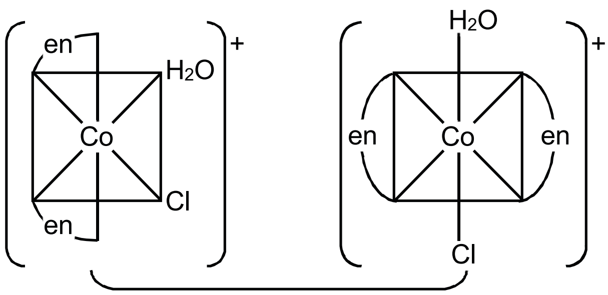
(1*) aquachlorobis(ethylenediamine)cobalt(II) chloride

(2) pentaaquachlorochromium(III)chloride

(3) potassium amminetrichloroplatinate(II)

(4) potassium tris(oxalato)chromate(III)

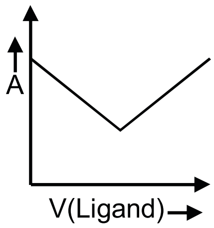
Sol. $[Co(H_2O)Cl(en)_2]Cl$

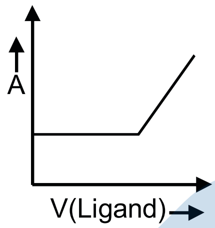


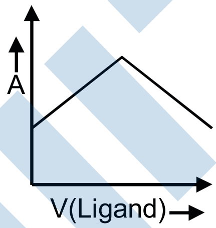
(Geometrical isomer)

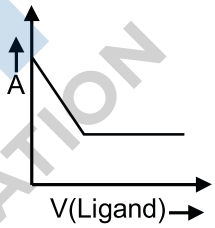
46. In a complexometric titration of metal ion with ligand

M(Metal ion)+L(Ligand) $\rightarrow$ C(Complex) end point is estimated spectrophotometrically (through light absorption). If 'M' and 'C' do not absorb light and only 'L' absorbs, then the titration plot between absorbed light (A) versus volume of ligand 'L' (V) would look like :

(1) 

(2*) 

(3) 

(4) 

Sol. Initially ligand consumed by metal due to formation of complex. So absorbed light (A) remain constant, after complex formation is completed, extra volume of ligand solution increases ligand concentration and also increases absorbed light.

47. In the extraction of copper from its sulphide ore, metal is finally obtained by the oxidation of cuprous sulphide with :

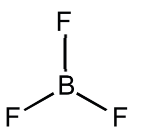
(1) Fe_2O_3 (2*) Cu_2O (3) SO_2 (4) CO

Sol. $Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2$

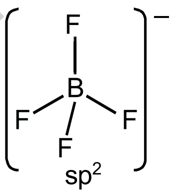
48. Which of the following conversions involves change in both shape and hybridisation ?

(1) $NH_3 \rightarrow NH_4^+$ (2) $CH_4 \rightarrow C_2H_6$ (3) $H_2O \rightarrow H_3O^+$ (4*) $BF_3 \rightarrow BF_4^-$

Sol. $BF_3 \longrightarrow BF_4^-$



sp^2
Triangle planer

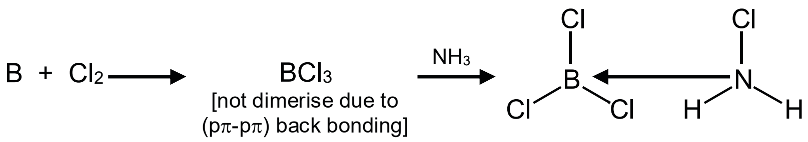


sp^2
Tetrahedral

49. A group 13 element 'X' reacts with chlorine gas to produce a compound XCl_3. XCl_3 is electron deficient and easily reacts with NH_3 to form $Cl_3X \leftarrow NH_3$ adduct; however, XCl_3 does not dimerize. X is :

(1*) B (2) Al (3) Ga (4) In

Sol. BCl_3

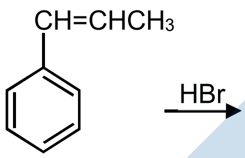


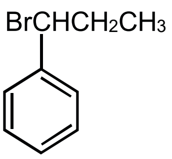
50. When XO_2 is fused with an alkali metal hydroxide in presence of an oxidizing agent such as KNO_3 ; a dark green product is formed which disproportionates in acidic solution to afford a dark purple solution. X is :

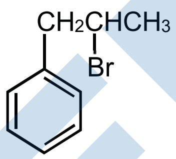
(1) Ti (2) V (3) Cr (4*) Mn

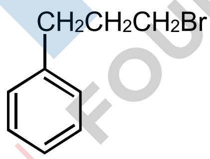
Sol. $MnO_2 + KOH \longrightarrow K_2MnO_4$ (dark green) $\xrightarrow[\text{solution}]{\text{Acidic}}$ $KMnO_4$ (dark purple)

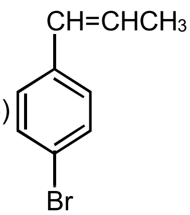
51. The major product of the following reaction is :

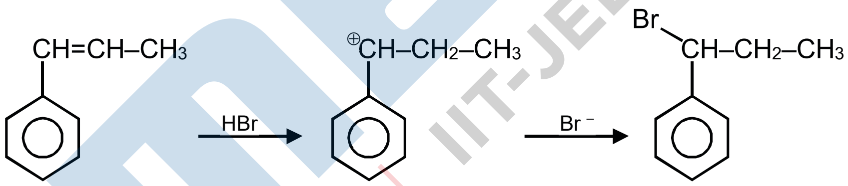


(1*) 

(2) 

(3) 

(4) 

Sol. 

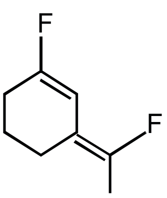
52. For standardizing NaOH solution, which of the following is used as a primary standard ?

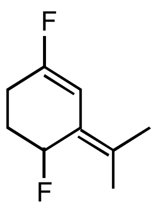
(1) Ferrous Ammonium Sulfate (2) dil. HCl

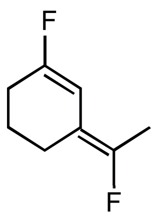
(3*) Oxalic acid (4) Sodium tetraborate

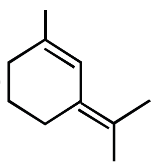
Sol. Oxalic acid is used as a primary standard for NaOH standardizing.

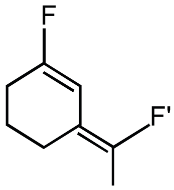
53. The most polar compound among the following is :

(1*) 

(2) 

(3) 

(4) 

Sol. In  the bond dipole vector of C–F bond is not subtractive.

54. The correct match between items of List - I and List - II is :

List - I		List - II	
(A)	Phenelzine	(P)	Pyrimidine
(B)	Chloroxylenol	(Q)	Furan
(C)	Uracil	(R)	Hydrazine
(D)	Ranitidine	(S)	Phenol

(1*) (A)-(R), (B)-(S), (C)-(P), (D)-(Q)
(2) (A)-(S), (B)-(R), (C)-(P), (D)-(Q)
(3) (A)-(S), (B)-(R), (C)-(Q), (D)-(P)
(4) (A)-(R), (B)-(S), (C)-(Q), (D)-(P)

Sol. → Phenelzine contains hydrazine

→ Chloroxylenol contains phenol

→ Uracil is the pyrimidine base

→ Ranitidine contains furan ring

55. Among the following, the incorrect statement is :

(1) Maltose and lactose has 1, 4-glycosidic linkage.

(2*) Sucrose and amylose has 1, 2-glycosidic linkage.

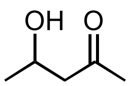
(3) Cellulose and amylose has 1, 4-glycosidic linkage.

(4) Lactose contains β-D-galactose and β-D-glucose.

Sol. In amylose 1,4-glycosidic linkage is present.

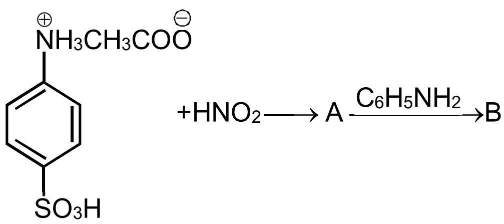
56. Which of the following compounds will most readily be dehydrated to give alkene under acidic condition?

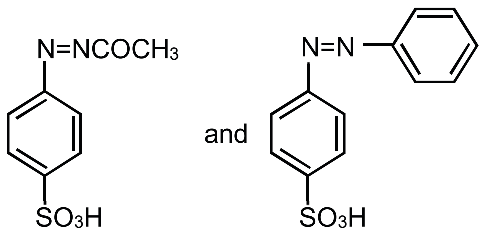
(1) 1-Pentanol
(2*) 4-Hydroxypentan-2-one
(3) 3-Hydroxypentan-2-one
(4) 2-Hydroxycyclopentanone

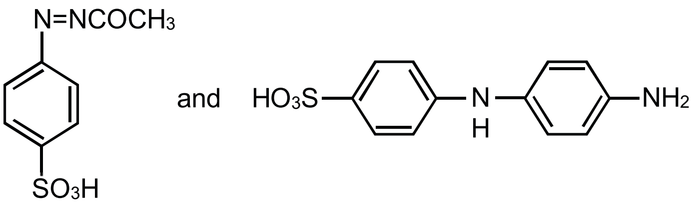
Sol. 

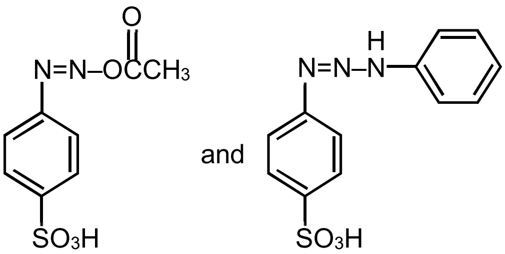
will most readily be dehydrated to give conjugated alkene.

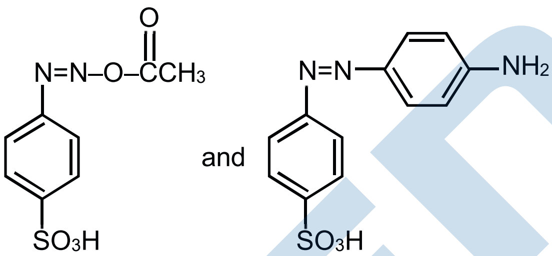
57. Products A and B formed in the following reactions are respectively :

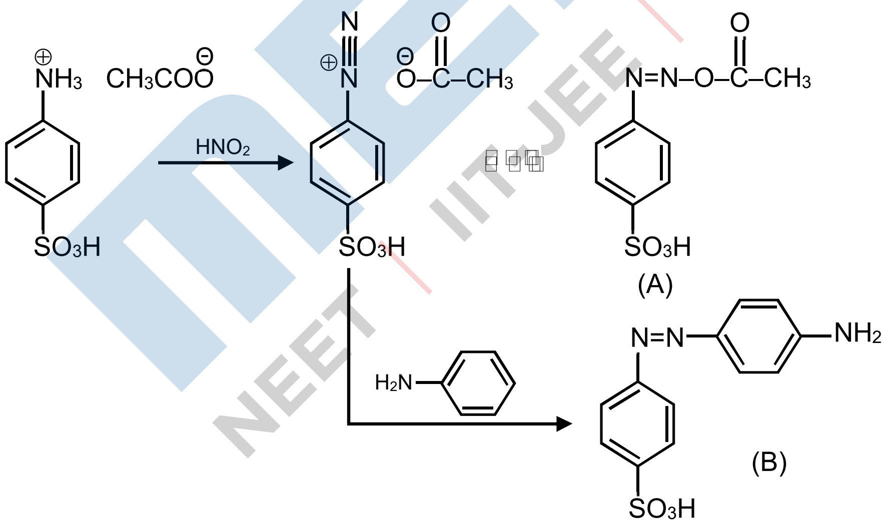


(1) 

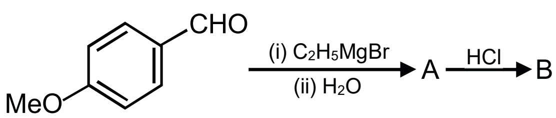
(2) 

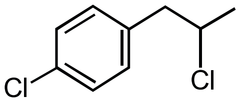
(3) 

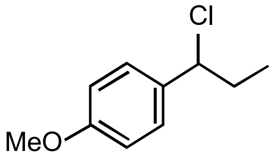
(4*) 

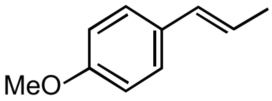
Sol. 

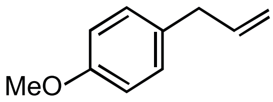
58. The major product B formed in the following reaction sequence is :

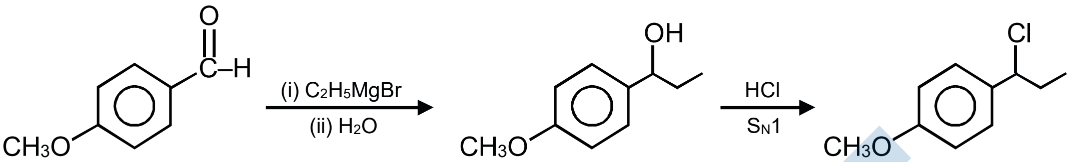


(1) 

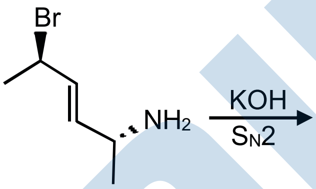
(2*) 

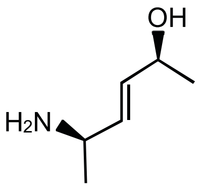
(3) 

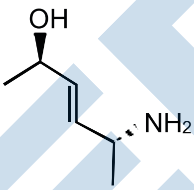
(4) 

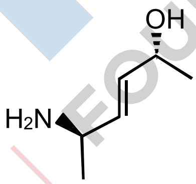
Sol. 

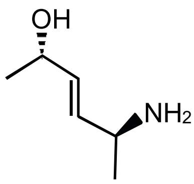
59. The major product of the following reaction is :

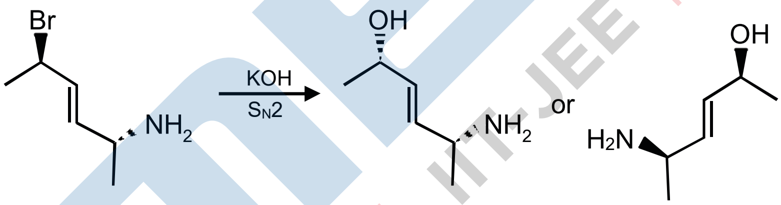


(1*) 

(2) 

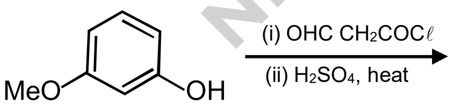
(3) 

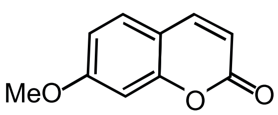
(4) 

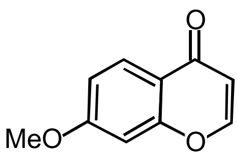
Sol. 

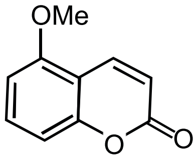
Inversion takes place at the carbon containing bromine atom.

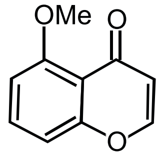
60. The major product of the following reaction is :

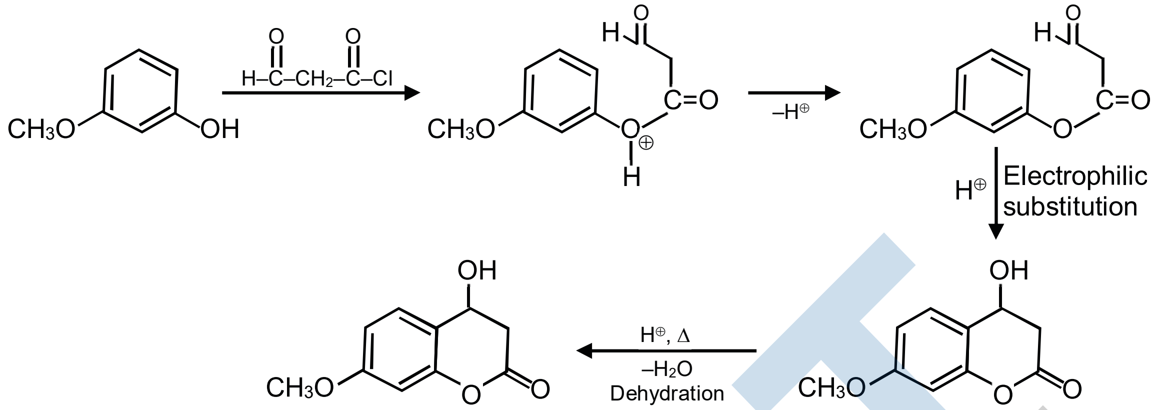


(1*) 

(2) 

(3) 

(4) 

Sol. 

PART-C-MATHEMATICS

61. Let N denote the set of all natural numbers. Define two binary relations on N as $R_1 = \{(x, y) \in N \times N :$ $2x + y = 10\}$ and $R_2 = \{(x, y) \in N \times N : x + 2y = 10\}$. Then

(1) Range of R_1 is {2, 4, 8}

(2*) Range of R_2 is {1, 2, 3, 4}

(3) Both R_1 and R_2 are symmetric relations.

(4) Both R_1 and R_2 are transitive relations.

Sol. $R_1 = \{(1, 8), (2, 6), (3, 4), (9, 2)\}$

$R_2 = \{(8, 1), (6, 2), (4, 3), (2, 4)\}$

Range of $R_2 = \{1, 2, 3, 4\}$

62. Let p, q and r be real numbers ($p \neq q$, $r \neq 0$), such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to

(1) $\frac{p^2 + q^2}{2}$ (2*) $p^2 + q^2$ (3) $2(p^2 + q^2)$ (4) $p^2 + q^2 + r^2$

Sol. $(2x + p + q) r = (x + p)(x + q)$

$x^2 + (p + q - 2r)x + pq - pr - qr = 0$

$p + q = 2r$ ....(i)

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 0 - 2[pq - pr - qr] = -2pq + 2r(p + q) = -2pq + (p + q)^2 = p^2 + q^2$

63. The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n$ = 1, is

(1) 2 (2*) 3 (3) 5 (4) 6

Sol. $\left(\frac{1+i\sqrt{3}}{n-i\sqrt{3}}\right)^n = 1$

$\left(\frac{-2\omega^2}{-2\omega}\right)^n = 1$

$\omega^n = 1$

least positive integer value of n is 3.

64. Let A = $\begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\1 & 1 & 1\end{bmatrix}$ and B = A^{20}. Then the sum of the elements of the first column of B is

(1) 210 (2) 211 (3*) 231 (4) 251

Sol. $A = \begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\1 & 1 & 1\end{bmatrix}$

$A^2 = \begin{bmatrix}1 & 0 & 0\\2 & 1 & 0\\3 & 2 & 1\end{bmatrix}$; $A^3 = \begin{bmatrix}1 & 0 & 0\\3 & 1 & 0\\6 & 3 & 1\end{bmatrix}$

$A^4 = \begin{bmatrix}1 & 0 & 0\\4 & 1 & 0\\10 & 4 & 1\end{bmatrix}$.......$A^{20} = \begin{bmatrix}1 & 0 & 0\\20 & 1 & 0\\210 & 20 & 1\end{bmatrix}$

Sum of the elements of first column = 231

65. The number of values of k for which the system of linear equations,

(k + 2) x + 10y = k

kx + (k + 3)y = k – 1

has no solution, is

(1*) 1 (2) 2 (3) 3 (4) infinitely many

Sol. For no solution

$\frac{k+2}{k} = \frac{10}{k+3} \neq \frac{k}{k-1}$

(k + 2) (k + 3) = 10 k

$k^2 - 5k + 6 = 0 \Rightarrow k = 2,3$

$k \neq 2$ for k = 2 both lines identical

so k = 3 only

so number of values of k is 1

66. The number of numbers between 2000 and 5000 that can be formed with the digits 0, 1, 2, 3, 4 (repetition of digits is not allowed) and are multiple of 3 is

(1) 24 (2*) 30 (3) 36 (4) 48

Sol. number can be formed y (0, 1, 2, 3) or (0, 2, 3, 4)

number of 4 digits number = 2 × 3 ! + 3 × 3 ! = 30

67. The coefficient of x^2 in the expansion of the product $(2 - x^2)\left((1+2x+3x^2)^6 + (1-4x^2)^6\right)$ is

(1) 107 (2*) 106 (3) 108 (4) 155

Sol. Coefficient of x^2 = 2 coefficient of x^2 in $((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$ – constant term

$$(1+2x+3x^2) = \sum_{r=0}^{6} {}^6C_r (2x+3x^2)^r$$

$= {}^6C_0 + {}^6C_1 (2x + 3x^2) + {}^6C_2 (2x + 3x^2)^2 + \ldots$

coefficient of x^2 = 2(18 + 60 – 24) – 2

= 108 – 2 = 106

68. Let $\frac{1}{x_1}, \frac{1}{x_2}, \ldots\ldots, \frac{1}{x_n}$ ($x_i \neq 0$ for i = 1, 2, ……., n) be in A.P. such that $x_1 = 4$ and $x_{21} = 20$.

If n is the least positive integer for which $x_n > 50$, then $\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)$ is equal to

(1) $\frac{1}{8}$ (2) 3 (3) $\frac{13}{8}$ (4*) $\frac{13}{4}$

Sol. $\frac{1}{4} + 20d = \frac{1}{20}$

$d = \frac{-1}{100}$

$\frac{1}{x_n} < \frac{1}{50}$

$\frac{1}{4} - \frac{n-1}{100} < \frac{1}{50} \quad \Rightarrow n > 24$

n = 25

$$\sum_{i=1}^{25}\left(\frac{1}{x_i}\right) = \frac{25}{2}\left[2 \times \frac{1}{4} - \frac{1}{100} \times 24\right] = \frac{13}{4}$$

69. The sum of the first 20 terms of the series $1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + \ldots\ldots$, is

(1*) $38 + \frac{1}{2^{19}}$ (2) $38 + \frac{1}{2^{20}}$ (3) $39 + \frac{1}{2^{20}}$ (4) $39 + \frac{1}{2^{19}}$

Sol. $1+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16}+........$

$= (2-1)+\left(2-\frac{1}{2}\right)+\left(2-\frac{1}{4}\right)+\left(2-\frac{1}{8}\right)+\left(2-\frac{1}{16}\right)+..........$ upto 20 terms

$= 40-\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+......\text{upto 20 terms}\right)$

$= 40-\left(\frac{1-\left(\frac{1}{2}\right)^{20}}{1-\frac{1}{2}}\right) = 40-2+\frac{1}{2^{19}} = 38+\frac{1}{2^{19}}$

70. $\lim\limits_{x\to 0}\frac{(27+x)^{\frac{1}{3}}-3}{9-(27+x)^{\frac{2}{3}}}$ equals

(1) $\frac{1}{3}$ (2) $\frac{-1}{3}$ (3*) $\frac{-1}{6}$ (4) $\frac{1}{6}$

Sol. $\lim\limits_{x\to 0}\frac{3\left[\left(1+\frac{x}{27}\right)^{\frac{1}{3}}-1\right]}{9\left[1-\left(1+\frac{x}{27}\right)^{\frac{2}{3}}\right]}$

$\lim\limits_{x\to 0}\frac{1}{3}\left[\frac{\frac{x}{81}}{-\frac{2}{3}\frac{x}{27}}\right] = \frac{-1}{6}$

71. If the function f defined as f(x) = $\frac{1}{x}-\frac{k-1}{e^{2x}-1}$, $x \neq 0$, is continuous at x = 0, then the ordered pair (k,f,(0)) is equal to

(1) (3, 2) (2*) (3, 1) (3) (2, 1) (4) $\left(\frac{1}{3}, 2\right)$

Sol. $f(x)=\frac{1}{x}-\frac{k-1}{e^{2x}-1};\ x\neq 0$

f(x) is continuous at x = 0

$\Rightarrow f(0)=\lim\limits_{x\to 0}\frac{1}{x}-\frac{k-1}{e^{2x}-1}$

$= \lim\limits_{x\to 0}\frac{(1+(2x)+\frac{1}{2!}(2x)^2+..........(-1-x(k-1)))}{2x^2\left(\frac{e^{2x}-1}{2x}\right)}$

Clearly k = 3 and f(0) = 1

72. If $x = \sqrt{2^{\operatorname{cosec}^{-1} t}}$ and $y = \sqrt{2^{\sec^{-1} t}}$ $(|t| \geq 1)$, then $\frac{dy}{dx}$ is equal to

(1) $\frac{y}{x}$ (2) $\frac{x}{y}$ (3*) $-\frac{y}{x}$ (4) $-\frac{x}{y}$

Sol. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{1}{2\sqrt{2^{\sec^{-1} t}}} 2^{\sec^{-1} t} \ln 2 \left(\frac{1}{t\sqrt{t^2-1}}\right)}{\frac{1}{2\sqrt{2^{\operatorname{cosec}^{-1} t}}} 2^{\operatorname{cosec}^{-1} t} \ln 2 \left(\frac{1}{t\sqrt{t^2-1}}\right)}$

$= -\frac{\sqrt{2^{\sec^{-1} t}}}{\sqrt{2^{\operatorname{cosec}^{-1} t}}} = -\frac{y}{x}$

73. Let M and m be respectively the absolute maximum and the absolute minimum values of the function, $f(x) = 2x^3 - 9x^2 + 12x + 5$ in the interval [0, 3]. Then M – m is equal to

(1) 5 (2*) 9 (3) 4 (4) 1

Sol. $f(x) = 2x^3 - 9x^2 + 12 + 5,\ x \in [0, 3]$

$f'(x) = 6x^2 - 18x + 12$

$f'(x) = 6(x-1)(x-2)$

$f(1) = 10$

$f(2) = 9$

$f(0) = 5$

$f(3) = 14$

M = 14, m – 5 $\Rightarrow$ M – m = 9

74. If $\int \frac{\tan x}{1 + \tan x + \tan^2 x} dx = x - \frac{K}{\sqrt{A}} \tan^{-1}\left(\frac{K \tan x + 1}{\sqrt{A}}\right)$ + C, (C is a constant of integration), then the ordered pair (K, A) is equal to

(1) (2, 1) (2) (– 2, 3) (3*) (2, 3) (4) (– 2, 1)

Sol. $I = \int \frac{\tan}{1 + \tan x + \tan^2 x} dx$

$\int \left(1 - \frac{\sec^2 x}{1 + \tan x + \tan^2 x}\right) dx$

$x = \int \frac{dt}{1 + t + t^2}$, where tanx = t $\Rightarrow$ $\sec^2 x\, dx = dt$

$= \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}} = x - \frac{1}{\sqrt{3}/2} \tan^{-1}\left(\frac{t + \frac{1}{2}}{\sqrt{3}/2}\right) + C = x - \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2\tan x + 1}{\sqrt{3}}\right) + C$

= k = 2, A = 3.

75. If $f(x) = \int_0^x t(\sin x - \sin t)dt$, then

(1) f '''(x) + f ''(x) = sin x
(2) f '''(x) + f ''(x) – f '(x) = cos x
(3*) f '''(x) + f '(x) = cos x – 2x sin x
(4) f '''(x) – f ''(x) = cos x – 2x sin x

Sol. $f(x) = \int_0^x t(\sin x - \sin t)dt$

$f(x) = \sin x\int_0^x tdt - \int_0^x t\sin tdt$

$f'(x) = (\sin x)x + \cos x\int_0^x tdt - x\sin x$

$f'(x) = \cos x\int_0^x tdt$

$f''(x) = (\cos x)x - (\sin x)\int_0^x tdt$

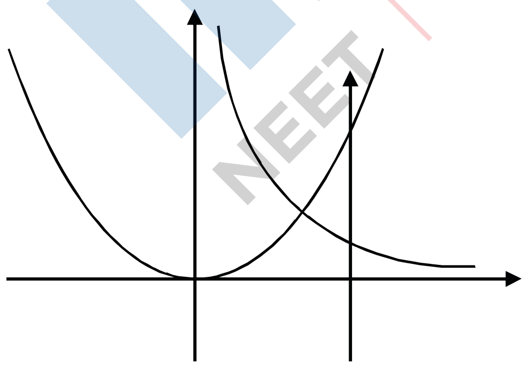
$f'''(x) = x(-\sin x) + \cos x - (\sin x)x - (\cos x)\int_0^x tdt$

f'''(x) + f'(x) = cosx – 2x sinx

76. If the area of the region bounded by the curves, $y = x^2$, $y = \frac{1}{x}$ and the lines y = 0 and x = t (t > 1) is 1 s unit, then t is equal to

(1) $e^{\frac{3}{2}}$
(2) $\frac{4}{3}$
(3) $\frac{3}{2}$
(4*) $e^{\frac{2}{3}}$

Sol. $\int_x^1 x^2dx + \int + \frac{1}{x}dx = 1$



$\frac{1}{3}x + \ln t = 1 \quad \Rightarrow \quad t = e^{\frac{2}{3}}$

77. The differential equation representing the family of ellipses having foci either on the x-axis or on the y-axis, centre at the origin and passing through the point (0, 3) is

(1) $xy\,y'' + x(y')^2 - y\,y' = 0$ (2) $x + y\,y'' = 0$

(3) $xy\,y' + y^2 - 9 = 0$ (4*) $xy\,y' - y^2 + 9 = 0$

Sol. Equation of ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Passes (0, 3) $\Rightarrow$ $\frac{x^2}{a^2}+\frac{y^2}{9}=1$ ....(1) $\frac{x}{a^2}=-\frac{y}{9}\frac{dy}{dx}$

$\frac{2x}{a^2}+\frac{2y}{9}\frac{dy}{dx}=0$ ....(2) $\frac{1}{a^2}=-\frac{y}{9x}y^1$

By (1) & (2) D. equation is

$$-\frac{xy}{9}y^1+\frac{y^2}{9}=1$$

$\Rightarrow$ $xy\,y^1 - y^2 + 9 = 0$

78. The locus of the point of intersection of the lines, $\sqrt{2}x - y + 4\sqrt{2}k = 0$ and $\sqrt{2}kx + ky - 4\sqrt{2} = 0$ (k is any non-zero real parameter), is

(1) an ellipse whose eccentricity is $\frac{1}{\sqrt{3}}$.

(2*) an ellipse with length of its major axis $8\sqrt{2}$,

(3) a hyperbola whose eccentricity is $\sqrt{3}$.

(4) a hyperbola with length of its transverse axis $8\sqrt{2}$,

Sol. $\sqrt{2}x - y + 4\sqrt{2}k = 0$ ....(i)

$\sqrt{2}kx + ky - 4\sqrt{2} = 0$ ....(ii)

Eliminating k by (i) and (ii)

$$\left(\sqrt{2}x+y\right)\left(\frac{\sqrt{2}x-y}{-4\sqrt{2}}\right)=4\sqrt{2}$$

$2x^2 - y^2 = -32$

$\frac{y^2}{32}-\frac{x^2}{16}=1$ Hyperbola

$e=\sqrt{1+\frac{16}{32}}=\sqrt{\frac{3}{2}}$ and length of transverse axis $= 8\sqrt{2}$

79. If a circle C, whose radius is 3, touches externally the circle, $x^2 + y^2 + 2x - 4y - 4 = 0$ at the point (2, 2), then the length of the intercept cut by this circle C, on the x-axis is equal to

(1*) $2\sqrt{5}$ (2) $3\sqrt{2}$ (3) $\sqrt{5}$ (4) $2\sqrt{3}$

Sol. Centre of given circle = (–1, 2)

and radius $= \sqrt{1+4+4} = 3$

centre of required circle (5,2)

or (–4,2)

length of intercept on x-axis will be square in both circle

so one required circle $(x-5)^2+(y-2)^2=32$

$x^2+y^2-10x-4y+20=0$

Length of x intercept $=2\sqrt{g^2-c}$

$=2\sqrt{25-20}=2\sqrt{5}$

80. Let P be a point on the parabola, $x^2 = 4y$. If the distance of P from the centre of the circle, $x^2 + y^2 + 6x + 8 = 0$ is minimum, then the equation of the tangent to the parabola at P, is

(1) $x + 4y - 2 = 0$ (2) $x - y + 3 = 0$ (3*) $x + y + 1 = 0$ (4) $x + 2y = 0$

Sol. Let P $(2t, t^2)$

equation normal at P to $x^2 = 4y$ be

$$y-t^2=-\frac{1}{t}(x-2t)$$

it passes through (–3, 0)

$$0-t^2=-\frac{1}{t}(-3-2t)$$

$t^3 + 2t + 3 = 0$

$(t + 1)(t^2 - 1 + 3) = 0$

$\Rightarrow t = -1$

Point P is (–2, 1)

equation of tangent to $x^2 = 4y$ at (–2, 1)

$x(-2) = 2(y + 1)$

$x + y + 1 = 0$

81. If the length of the latus rectum of an ellipse is 4 units and the distance between a focus and its nearest vertex on the major axis is $\frac{3}{2}$ units, then its eccentricity is

(1) $\frac{1}{2}$ (2*) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{1}{9}$

Sol. $\frac{2b^2}{a}=4 \quad \Rightarrow \quad b^2 = 2a$

$$b^2=a^2(1-e^2), a(1-e)=\frac{3}{2}$$

$2 = a(1 - e)(1 + e)$

$$2=\frac{3}{2}(1+e) \qquad e=\frac{1}{3}$$

82. The sum of the intercepts on the coordinate axes of the plane passing through the point (– 2, – 2, 2) and containing the line joining the points (1, – 1, 2) and (1, 1, 1) is

(1) 4 (2*) – 4 (3) – 8 (4) 12

Sol. Equation plane

$$\begin{vmatrix} x+2 & y+2 & z-2 \\ -3 & -1 & 0 \\ -3 & -3 & 1 \end{vmatrix} = 0$$

$\Rightarrow -(x + 2) + 3(y + 2) + 6(z - 2) = 0$

$\Rightarrow x - 3y - 6z + 8 = 0$

sum of intercepts $= -8 + \frac{8}{3} + \frac{8}{6} = -4$

83. If the angle between the lines, $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{5-x}{-2} = \frac{7y-14}{p} = \frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then p is equal to

(1*) $\frac{7}{2}$ (2) $\frac{2}{7}$ (3) $-\frac{7}{4}$ (4) $-\frac{4}{7}$

Sol. $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x-5}{2} = \frac{y-2}{P/7} = \frac{z-3}{4}$

Angle between both lines is $\cos^{-1}\left(\frac{2}{3}\right) = \cos^{-1}\left(\frac{4 + \frac{2P}{7} + 4}{3\sqrt{4 + \frac{P^2}{49} + 16}}\right)$

$$\Rightarrow \frac{2}{3} = \frac{56 + 2P}{3\sqrt{P^2 + 980}} \Rightarrow \sqrt{P^2 + 980} = P + 28 \Rightarrow P^2 + 980 = P^2 + 56P + 784 \Rightarrow 56P = 196 \Rightarrow P = \frac{7}{2}$$

84. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$ and a vector $\vec{b}$ be such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \times \vec{b} = 3$, Then $|\vec{b}|$ equals

(1) $\frac{11}{3}$ (2) $\frac{11}{\sqrt{3}}$ (3*) $\sqrt{\frac{11}{3}}$ (4) $\frac{\sqrt{11}}{3}$

Sol. $\vec{a} \times \vec{b} = \vec{c}$

$\vec{a} \times (\vec{a} \times \vec{b}) = \vec{a} \times \vec{c}$

$(\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} = \vec{a} \times \vec{c}$

$3\vec{a} - 3\vec{b} = -2\hat{i} + \hat{j} + \hat{k}$

$3\hat{i} + 3\hat{j} + 3\hat{k} - 3\vec{b} = -2\hat{i} + \hat{j} + \hat{k}$

$\vec{b} = \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$

$$|\vec{b}| = \frac{\sqrt{25+4+4}}{3}$$

$$|\vec{b}| = \sqrt{\frac{11}{3}}$$

85. The mean and the standard deviation (s.d.) of five observations are 9 and 0, respectively. If one of the observations is changed such that the mean of the new set of five observation becomes 10, then their s.d. is

(1*) 0 (2) 1 (3) 2 (4) 4

Sol. Standard deviations with be same

so S.D is 0

86. Let A, B and C be three events, which are pair-wise independent and $\bar{E}$ denotes the complement of an event E. If P(A ∩ B ∩ C) = 0 and P(C) > 0, then $P\left[\left(\bar{A} \cap \bar{B}\right)\middle| C\right]$ is equal to

(1*) $P(\bar{A}) - P(B)$ (2) $P(A) + P(\bar{B})$ (3) $P(\bar{A}) - P(\bar{B})$ (4) $P(\bar{A}) + P(\bar{B})$

Sol. $P\left[\bar{A} \cap \bar{B} \mid C\right] = \dfrac{P\left[\left(\bar{A} \cup \bar{B}\right) \cap C\right]}{P(C)}$

$$= \frac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$$

$$= \frac{P(C) - P(A)P(C) + P(B)P(C)}{P(C)}$$

= 1 – P(A) – P(B)

$= P\left(\bar{A}\right) - P(B)$ or $P\left(\bar{B}\right) - P(A)$

87. Two different families A and B are blessed with equal number of children. There are 3 tickets to be distributed amongst the children of these families so that no child gets more than one ticket. If the probability that all the tickets go to the children of the family B is $\frac{1}{12}$, then the number of children in each family is

(1) 3 (2) 4 (3*) 5 (4) 6

Sol. Let n number of children are there in each family

$$\frac{1}{12} = \frac{{}^{n}C_3 . 3!}{{}^{2n}C_3 . 3!}$$

$$\frac{{}^{n}C_3}{{}^{2n}C_3} = \frac{1}{12} \quad n = 5$$

88. If an angle A of a ΔABC satisfies 5cos A + 3 = 0, then the roots of the quadratic equation, $9x^2 + 27x + 20 = 0$ are

(1) sec A, cot A (2) sin A, sec A (3*) sec A, tan A (4) tan A, cos A

Sol. $5\cos A + 3 = 0 \Rightarrow \cos A = -\frac{3}{5}$ clearly A $\in$ (90°, 180°)

Now roots of equation 9x2 + 27x + 20 = 0 are $= -\frac{5}{3}$ and $-\frac{4}{3}$

$\Rightarrow$ Roots secA and tanA

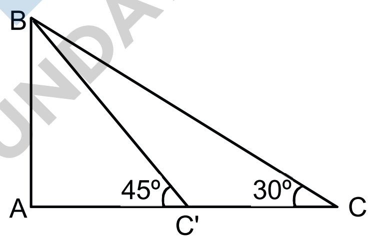
89. A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizontal road. If it takes 18 min. for the angle of depression of the car to change from 30° to 45°; then after this, the time taken (in min.) by the car to reach the foot of the tower, is

(1*) $9(1+\sqrt{3})$ (2) $18(1+\sqrt{3})$ (3) $18(\sqrt{3}-1)$ (4) $\frac{9}{2}(\sqrt{3}-1)$

Sol. Let length of tower = h

$\Rightarrow$ AC' = AB = h

and $AC = AB\cot 30° = \sqrt{3}h \quad \Rightarrow \quad CC' = (\sqrt{3}-1)h$



Time taken by car form C to C' = 18 min

$\Rightarrow$ time take by car to reach the foot of the tower $= \frac{18}{\sqrt{3}-1}$ min.

$= 9(\sqrt{3}+1)$min

90. If $p \to (\sim p \vee \sim q)$ is false, then the truth values of p and q are respectively

(1) F, F (2) T, F (3) F, T (4*) T, T

Sol. $P \to (\sim p \vee \sim q)$

p	q	$\sim p \vee \sim q$	$p \to (\sim p \vee \sim q)$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T